Record Statistics of Continuous Time Random Walk

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Abstract. - The statistics of records for a time series generated by a continuous time random walk is studied, and found to be independent of the details of the jump length distribution, as long as the latter is continuous and symmetric. However, the statistics depend crucially on the nature of the waiting time distribution. The probability of finding M records within a given time duration t, for large t, has a scaling form, and the exact scaling function is obtained in terms of the one-sided Lévy stable distribution. The mean of the ages of the records, defined as $\langle t/M\rangle$, differs from $t/\langle M\rangle$. The asymptotic behaviour of the shortest and the longest ages of the records are also studied.

People's interest in records is evident from the popularity of the *Guinness World Records* — which itself holds a record as the best selling copyright book. An observation is called a (upper) record if its value exceeds that of all previous observations. Applications of records are found in diverse fields such as meteorology [1], hydrology [2], economics [3], and sports [4]. Records also play an important role in highlighting climate data to increase public awareness of climate change and global warming — e.g., a recent analysis from the United States *National Climatic Data Center* reveals that the global combined land and ocean average surface temperatures for June, April to June average, and January to June average, of 2010, are the warmest on record [5].

Frequently asked questions in the study of records include: (a) How many records occur in a given duration? (b) How long does a record stay before it is broken by a new one? In the case of a discrete time series $\{x_0, x_1, x_2, \ldots, x_N\}$ consisting of independent and identically distributed (IID) random variables, the statistics regarding the above questions are well-studied [6–9] — which have been useful in understanding and analysing evolutions in complex systems, ranging from phase slip in charge density waves, ageing in glassy systems, to biological macroevolution [10] and adaptation [11, 12]. The record statistics for independent but non-identically distributed entries have been studied recently in the context of biological evolution and warming climate [13–15].

In spite of the continued interest in records, the theory has been mostly restricted to the case of independent random variables — while a time series often contains correlated entries. It is only recently that Majumdar and Ziff [16] initiated the study of record statistics for correlated entries — by considering a time series generated by a "discrete time random walk" (DTRW): $x_i = x_{i-1} + \xi_i$ with IID jump lengths $\{\xi_i\}$. A subsequent study of record statistics of a random walk with Cauchy distributed jumps and a drift has appeared in the context of a driven particle in a random landscape [17]. These studies, however, assume that the samples are collected at regular intervals, say τ_0, so that the number of entries N, in a given time duration t, is fixed: $N = t/\tau_0$. This is not the case in many real situations where the events happen at irregular intervals. Consequently for a fixed time duration t, the number of entries in the time series $\{x(t_0), x(t_1), x(t_2), \cdots, x(t_N)\}$ is random and, therefore, the results for fixed N would not hold. Thus, the natural first step would be to study the statistics of records in the simplest possible model of correlated time series in continuous time. In this Letter we carry out this important step.

The well-known model that takes into account of the random waiting times $\tau_i = t_i - t_{i-1}$, between successive steps of a random walk, is the so-called "continuous time random walk" (CTRW), that was introduced by Montroll and Weiss [18], and ever since has been successfully used to describe anomalous diffusion in various complex systems [19–23], including finance and economics [24]. In this Letter, we obtain exact asymptotic results for the statistics of the number and the ages of records, of a time series generated by the CTRW with the IID waiting times

drawn from a probability density function (PDF) $\rho(\tau)$, and IID jump lengths drawn from a continuous and symmetric PDF $\phi(\xi)$. The results are independent of $\phi(\xi)$ — and the reason for this universality is the same as in Ref. [16], namely, the use of the Sparre-Andersen theorem [25,26]. However, with respect to the nature of $\rho(\tau)$, the statistics of records within a given time duration t, display rather rich scaling behaviour for large t. They are essentially determined by the behaviour of the Laplace transform $\widetilde{\rho}(s) = \int_0^\infty e^{-s\tau}\rho(\tau)\,d\tau = 1-(\tau_0 s)^\alpha + \cdots$ as $s\to 0$, where $\alpha=1$ as long as the mean waiting time is finite, whereas the latter becomes infinite for $0<\alpha<1$. The asymptotic results of Ref. [16] correspond to the special case $\alpha=1$ of the more general model considered in this Letter.

We first summarise our main results. We find that, the probability $P(M,t)$ of finding M records within a given time duration t, for large M and t with the scaled variable $M(t/\tau_0)^{-\alpha/2}$ fixed, has the scaling form

$$P(M,t) \sim (t/\tau_0)^{-\alpha/2}\, g_\alpha\Big(M(t/\tau_0)^{-\alpha/2}\Big). \tag{1}$$

The scaling function is given by

$$g_\alpha(x) = (2/\alpha)x^{-(1+2/\alpha)}L_{\alpha/2}(x^{-2/\alpha}), \quad 0<\alpha\le 1, \tag{2}$$

where $L_\mu(x)$ is the one-sided ($\beta=+1$) Lévy stable PDF. The asymptotic behaviour of the moments of the number of records is given by

$$\langle M^\nu\rangle \sim \frac{(2/\alpha)\Gamma(\nu)}{\Gamma(\nu\alpha/2)}\left(\frac{t}{\tau_0}\right)^{\nu\alpha/2}. \tag{3}$$

The mean of the ages of the records $\langle l\rangle = \langle t/M\rangle$, grows as

$$\left\langle\frac{t}{M}\right\rangle \sim \frac{\tau_0(\alpha/2)}{\Gamma\big(1-\frac{\alpha}{2}\big)}\left[\ln\Big(\frac{t}{\tau_0}\Big)-\Psi\Big(1-\frac{\alpha}{2}\Big)\right]\left(\frac{t}{\tau_0}\right)^{1-\alpha/2} \tag{4}$$

where $\Psi(x)=\Gamma'(x)/\Gamma(x)$ is the digamma function. On the other hand, $t/\langle M\rangle \sim \tau_0\Gamma(1+\alpha/2)(t/\tau_0)^{1-\alpha/2}$. As for the extreme ages of the records: the mean shortest age grows as $\langle l_{\min}\rangle \sim \tau_0\big[\Gamma(1-\alpha/2)\big]^{-1}(t/\tau_0)^{1-\alpha/2}$ whereas the mean longest age has a linear growth $\langle l_{\max}\rangle \sim C_\alpha\, t$ with the growth constant given by

$$C_\alpha = \int_0^\infty dx\left[1+x^{\alpha/2}e^x\int_0^x y^{-\alpha/2}e^{-y}\,dy\right]^{-1}. \tag{5}$$

Now we proceed to obtain the joint probability distribution of the ages and the number of records in a given duration t — from which the statistics of individual variables can be computed by integrating out the rest. In this context it is useful to refer to Fig. 1. Consider a time series $\{x(0),x(t_1),x(t_2),\cdots,x(t_N)\}$ generated by a CTRW with the IID jump sizes $\xi_i = x(t_i)-x(t_{i-1})$ drawn from a continuous and symmetric PDF $\phi(\xi)$, and the IID waiting times $\tau_i = t_i - t_{i-1}$ between successive jumps drawn from a one-sided PDF $\rho(\tau)$. We have set the initial time $t_0=0$, without loss of generality. Clearly, the total number of steps N, taken within a fixed time duration t — such that $t_N < t < t_{N+1}$ — is a random variable that varies from one realisation to another. Now for any given realisation of the walk, an entry $x(t_i)$ is a record iff $x(t_i) > x(t_j)$ for all $j<i$. Let R_n and T_n denote the value and the time of occurrence, respectively, of the n-th record. By convention, the first entry is taken to be the first record, i.e., $R_1 = x(0)$, and $T_1=0$. Evidently, $R_{n+1} > R_n$ and $T_{n+1} > T_n$ for all $n\ge 1$. Moreover, during the time interval between any two successive records, say the n-th and the $(n+1)$-th, the random walk does not exceed the earlier record value R_n. Therefore, the $(n+1)$-th record is the event in which a random walk starting from the position R_n at time T_n, exceeds its starting position R_n at time T_{n+1} for the first time. It is evident that the PDF of the above first-passage process — let it be denoted by $f(l)$ — depends neither on the actual value R_n nor on the individual times T_n and T_{n+1}, but solely on the time difference $l = T_{n+1}-T_n$. Let M denote the total number of records in the interval $[0,t]$. Then, after the time T_M, the random walk does not exceed the value R_M up to t, since there are no records in the interval $(T_M,t]$. The probability of not exceeding the starting position R_M solely depends on the time duration $l = t-T_M$. Let $q(l)$ denote this survival probability. Let l_n be the age of the n-th record, which is defined as: $l_n = T_{n+1}-T_n$ for $1\le n<M$, and $l_M = t-T_M$. Now, due to the Markov nature of the random walk, different inter-record intervals are uncorrelated. Thus, the joint distribution of the ages $\{l_i\}\equiv\{l_1,l_2,\ldots,l_M\}$ and the number M of records can be written as

$$P(\{l_i\},M,t) = \left[\prod_{i=1}^{M-1} f(l_i)\right] q(l_M)\,\delta\left(t-\sum_{i=1}^{M} l_i\right), \tag{6}$$

where the δ-function ensures that the ages add up to the total time duration t.

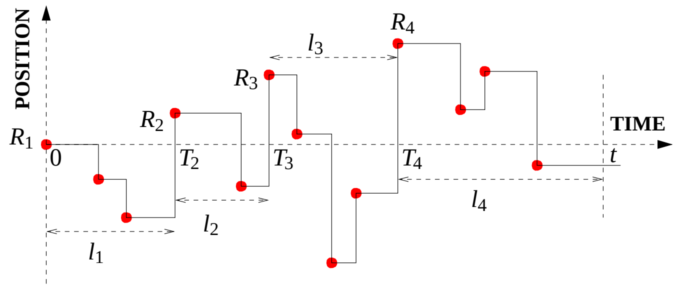

Fig. 1: (colour on-line). A realisation of a CTRW in the time interval $[0,t]$. Filled circles (in red) show the positions of the walker immediately after the jump. The horizontal lines between successive steps show the waiting times, whereas the vertical lines show the step sizes. R_i and T_i denote respectively the value and the time of occurrence of the i-th record. l_i denotes the age of the i-th record. The number of records $M=4$ for this particular realisation.

The first-passage properties of the CTRW are known to be related to that of the DTRW [18]. Let f_n be the probability that a random walk exceeds its starting position for the first time at the n-th step. Then $f(l) = \sum_{n=1}^{\infty} f_n\, p_n(l)$, where $p_n(l) = \int_0^\infty d\tau_1 \cdots \int_0^\infty d\tau_n \left[\prod_{i=1}^{n} \rho(\tau_i)\right] \delta(l - \sum_{i=1}^{n} \tau_i)$, is the PDF for the occurrence of the n-th step at time l. It is convenient to take a Laplace transform, which yields $\widetilde{f}(s) = \int_0^\infty e^{-sl} f(l)\, dl = \sum_{n=1}^{\infty} f_n\, [\widetilde{\rho}(s)]^n$. Similarly $q(l)$ can be related to the probability q_n that the random walk does not exceed its starting position up to the step n. Unlike the first-passage event, in this case the n-th step could occur at any time τ in the interval $(0, l)$, and then the walker does not move for the remaining duration $\tau_{n+1} = l - \tau$ (cf. Fig. 1). The probability that the walker remains fixed at least for a duration of τ_{n+1} is equal to $\int_{\tau_{n+1}}^{\infty} \rho(\tau)\, d\tau$ — which has the Laplace transform $s^{-1}[1 - \widetilde{\rho}(s)]$. Therefore, the Laplace transform of $q(l)$ reads: $\widetilde{q}(s) = s^{-1}[1 - \widetilde{\rho}(s)] \sum_{n=0}^{\infty} q_n [\widetilde{\rho}(s)]^n$. Now, as long as the PDF $\phi(\xi)$ of the jump lengths is continuous and symmetric, according to the Sparre-Andersen theorem [25, 26], both f_n and q_n are independent of $\phi(\xi)$ — the respective generating functions read: $\sum_{n=1}^{\infty} f_n z^n = 1 - \sqrt{1-z}$ and $\sum_{n=0}^{\infty} q_n z^n = 1/\sqrt{1-z}$. These yield the Laplace transforms:

$$\widetilde{f}(s) = 1 - \sqrt{1 - \widetilde{\rho}(s)} \quad \text{and} \quad \widetilde{q}(s) = s^{-1}\left[1 - \widetilde{f}(s)\right]. \tag{7}$$

It is now evident that the joint distribution given by Eq. (6), depends only on the waiting time distribution $\rho(\tau)$, and not the jump distribution $\phi(\xi)$.

Let us now compute the probability distribution of the number of records by integrating out the ages in Eq. (6), i.e., $P(M,t) = \int_0^\infty dl_1 \cdots \int_0^\infty dl_M \;\, P(\{l_i\}, M, t)$. In this regard, it is convenient take a Laplace transform with respect to t, that disentangles the global constraint imposed by the δ-function. This yields

$$\int_0^\infty P(M,t)\, e^{-st}\, dt = \frac{1 - \widetilde{f}(s)}{s} \left[\widetilde{f}(s)\right]^{M-1}, \tag{8}$$

for $M = 1, 2, \ldots, \infty$. The normalisation $\sum_M P(M,t) = 1$ can be readily verified by summing the above equation over M.

The behaviour of $P(M,t)$ for large t, can be extracted from the small s behaviour of Eq. (8) — which, according to Eq. (7), points to the small s behaviour of the Laplace transform $\widetilde{\rho}(s)$. Now, as long as the mean waiting time is finite — i.e., $\rho(\tau)$ decays faster than the power-law tail τ^{-2} at large τ, — the Laplace transform behaves as $\widetilde{\rho}(s) = 1 - \tau_0 s + \cdots$ as $s \to 0$. On the other hand, if waiting time distribution has a slower decay, $\rho(\tau) \sim \tau^{-(1+\alpha)}$ at large τ, with $0 < \alpha < 1$, the mean waiting time is infinite, and the Laplace transform $\widetilde{\rho}(s) = 1 - (\tau_0 s)^\alpha + \cdots$ as $s \to 0$. Considering both the above cases together, from Eq. (7) we get, $\widetilde{f}(s) \approx 1 - (\tau_0 s)^{\alpha/2}$ as $s \to 0$, where $0 < \alpha \leq 1$. Thus, taking the limit of small s and large M with keeping $M s^{\alpha/2}$ fixed in Eq. (8), results in $\int_0^\infty P(M,t)\, e^{-st}\, dt \approx \tau_0 (\tau_0 s)^{\alpha/2 - 1} e^{-M(\tau_0 s)^{\alpha/2}}$. This equation suggests that in the scaling limit $M \to \infty$, $t \to \infty$ while the scaled variable $M(t/\tau_0)^{-\alpha/2}$ is kept fixed, $P(M,t)$ should have the scaling form given by Eq. (1), so that

$$\int_0^\infty e^{-sy} \left[y^{-\alpha/2} g_\alpha\big(y^{-\alpha/2}\big)\right] dy = s^{\alpha/2 - 1} e^{-s^{\alpha/2}}. \tag{9}$$

To evaluate the scaling function $g_\alpha(x)$, it is useful to note that e^{-s^μ} is the Laplace transform of the one-sided Lévy stable PDF $L_\mu(y)$ [19]. From this, it immediately follows by differentiation that $\int_0^\infty e^{-sy}\, [y\, L_\mu(y)]\; dy = \mu s^{\mu-1} e^{-s^\mu}$. Comparing this equation with Eq. (9), the scaling function can be expressed in terms of the one-sided Lévy stable PDF as in Eq. (2). The normalisation of the PDF of the scaled number of records $x = M(t/\tau_0)^{-\alpha/2}$ can be checked by integrating Eq. (2) with the change of variable $y = x^{-2/\alpha}$ and subsequently noting that $L_\mu(y)$ is normalised to unity, i.e., $\int_0^\infty g_\alpha(x)\, dx = \int_0^\infty L_{\alpha/2}(y)\, dy = 1$. Incidentally, the PDF of the maximum displacement of a CTRW in a time interval t, also has a scaling form similar to Eq. (1), with the scaling function being identical to Eq. (2) *provided that the jumps are distributed according to a narrow distribution having a finite variance* [27]. However, it is important to realise that the record statistics of the CTRW is completely independent of the jump distribution as long as the PDF $\phi(\xi)$ of the jump lengths is continuous and symmetric —which also includes Lévy flights where $\phi(\xi) \sim |\xi|^{-1-\mu}$ is power-law distributed for large $|\xi|$ with $0 < \mu \leq 2$ and thus has a divergent second moment.

In general, $L_\mu(y)$ in Eq. (2) does not have a closed-form expression. However, expressing the right hand side of Eq. (9) as a series in s and then evaluating the inverse Laplace transform of the series, term by term, one gets

$$g_\alpha(x) = \frac{1}{\pi} \sum_{k=1}^{\infty} \frac{(-x)^{k-1}}{(k-1)!}\, \Gamma\Big(k\frac{\alpha}{2}\Big)\, \sin\Big(k\frac{\alpha}{2}\pi\Big). \tag{10}$$

This series representation is particularly useful for the numerical evaluation of $g_\alpha(x)$. Using the behaviour of $L_\mu(y)$ for small y [19], we find that for large x,

$$g_\alpha(x) \approx \frac{(\frac{\alpha}{2}x)^{-\frac{(1-\alpha)}{(2-\alpha)}}}{\sqrt{(2-\alpha)\pi}} \exp\left[-\left(\frac{2}{\alpha} - 1\right)\left(\frac{\alpha}{2}x\right)^{\frac{2}{(2-\alpha)}}\right]. \tag{11}$$

Figure 2 compares the analytical $g_\alpha(x)$ with the numerically obtained densities of the scaled variable $M(t/\tau_0)^{-\alpha/2}$ for $\alpha = 2/3$ and $\alpha = 1$. In fact, for these particular values of α, one has the explicit forms: $g_{2/3}(x) = (\sqrt{x}/\pi) K_{1/3}\big(2(x/3)^{3/2}\big)$ where $K_\nu(z)$ is the modified Bessel function, and $g_1(x) = e^{-x^2/4}/\sqrt{\pi}$ respectively. The latter agrees with the scaling function found in Ref. [16] for the DTRW, as expected. Numerical simulations of the CTRW are performed with both Gaussian and Cauchy distributed jumps to show the universality with respect to

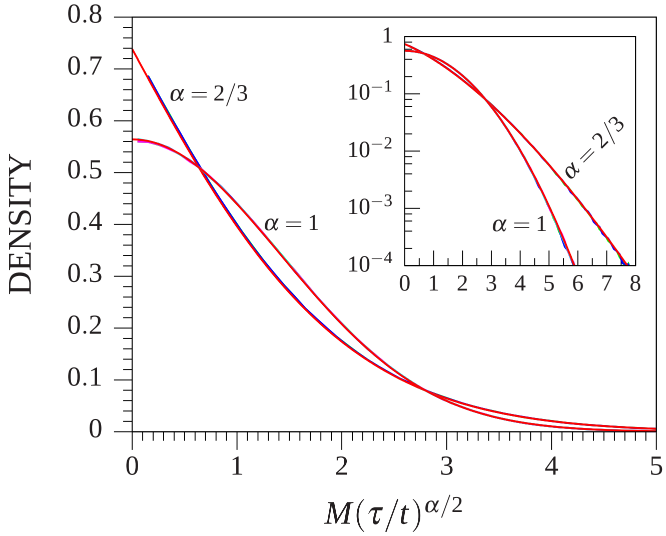

Fig. 2: (colour on-line). Density of the scaled records. For $\alpha = 2/3$, there are three curves that lie on top of each other: two of them (in green and blue) are obtained from numerical simulations (see text) and the third one (in red) plots the theoretical $g_{2/3}(x)$. For $\alpha = 1$ there are five curves: four (in green, blue, magenta and cyan) from the simulation (see text) and one (in red) plots $g_1(x)$. Inset displays the curves in semi-log scale.

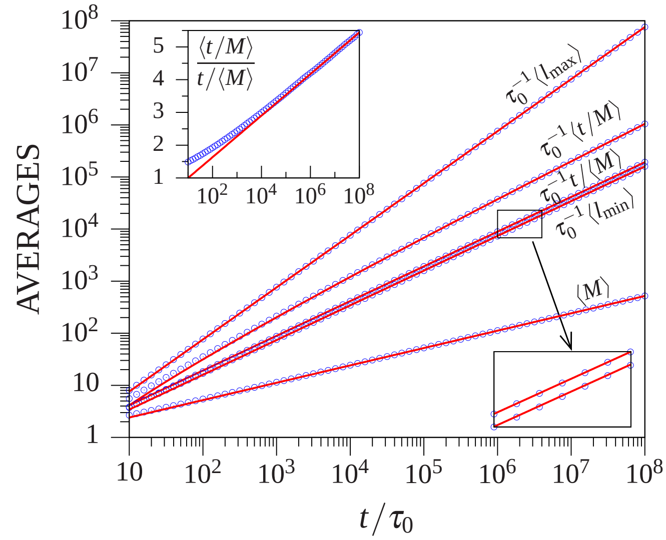

Fig. 3: (colour on-line). Different averages that are indicated on the curves, for $\alpha = 2/3$. The points (in blue) are obtained from numerical simulations by averaging over 10^6 realisations of CTRW. The lines (in red) plot the analytical asymptotic results reported in the text. Upper inset plots the ratio of $\langle t/M \rangle$ to $l/\langle M \rangle$. Lower inset zooms the portion shown.

the jump distributions. For $\alpha = 2/3$, the waiting times are drawn from a power-law tail $\rho(\tau) \sim \tau^{-5/3}$. For $\alpha = 1$, simulations are performed with both power-law $\rho(\tau) \sim \tau^{-5/2}$ and exponential $\rho(\tau) = e^{-\tau}$ distributions of the waiting times to show the universality of the results for finite mean waiting time. Each simulation is performed with 10^7 realisations of the CTRW, while taking $(t/\tau_0) = 10^7$ for $\alpha = 2/3$ and $(t/\tau_0) = 10^5$ for $\alpha = 1$.

To derive the moments $\langle M^\nu \rangle = \sum_M M^\nu P(M,t)$ for large t, we use the scaling form given by Eq. (1), and subsequently replace the sum over M by an integral over the scaled variable, which is justified for large t. This gives $\langle M^\nu \rangle \approx A_{\alpha,\nu}\,(t/\tau_0)^{\nu\alpha/2}$ with $A_{\alpha,\nu} = \int_0^\infty x^\nu g_\alpha(x)\,dx$. Now substituting $g_\alpha(x)$ from Eq. (2) and then making a change of variable $y = x^{-2/\alpha}$ we get $A_{\alpha,\nu} = \int_0^\infty y^{-\nu\alpha/2} L_{\alpha/2}(y)\,dy = (2/\alpha)\Gamma(\nu)\big[\Gamma(\nu\alpha/2)\big]^{-1}$, where in the last part we have used the known expression [19] for the negative moments of $L_\mu(y)$.

Let us now turn our attention to the statistics related to the ages of the records. The mean of the ages of the records for a given time interval t, is $\langle l \rangle = \langle M^{-1} \sum_{i=1}^M l_i \rangle = \langle t/M \rangle$. To compute this average, it is convenient to consider the moments generating function of M. Multiplying both sides of Eq. (8) by z^{M-1} and summing over M gives $\int_0^\infty dt\, e^{-st} \sum_{m=1}^\infty z^{M-1} P(M,t) = s^{-1}\big[1-\widetilde{f}(s)\big]\,\big[1 - z\widetilde{f}(s)\big]^{-1}$. Now integrating over z in $[0,1]$, and differentiating with respect to s, one gets

$$\int_0^\infty dt\, e^{-st} \left\langle \frac{t}{M} \right\rangle = \frac{\partial}{\partial s} \left\{ \frac{1-\widetilde{f}(s)}{s\widetilde{f}(s)}\, \ln\Big[1-\widetilde{f}(s)\Big] \right\}. \quad (12)$$

The asymptotic behaviour of the mean of the ages, for large t, emerges from the singularities near $s = 0$ in the above equation. The inverse Laplace transform of Eq. (12), with $\big[1 - \widetilde{f}(s)\big] \to (\tau_0 s)^{\alpha/2}$ and $\widetilde{f}(s) \to 1$, yields the result displayed by Eq. (4). It is interesting to note that $\langle t/M \rangle$ given by Eq. (4) differs from $t/\langle M \rangle \sim \tau_0 \Gamma(1+\alpha/2)(t/\tau_0)^{1-\alpha/2}$, where in the latter we have used first moment from Eq. (3). This non-self-averaging behaviour can be traced back to the broad distribution $f(l) \sim l^{-(1+\alpha/2)}$ of the inter-record intervals. It should be emphasised that the mean of the ages of the records in a given time interval t, is to be distinguished from the mean inter-record interval — in fact, the latter is infinite. The distinction between the two means arises from the definition of the age of the last record l_M in a given time interval t (see Fig. 1).

The statistics of the longest age of the records is similar to that of the longest excursion in a renewal process that was studied recently [28]. Therefore, skipping details, we cast the result in our context. The mean longest age of records in a given time duration t, for large t, reads $\langle l_{\max} \rangle \sim C_\alpha\, t$, where C_α is given by Eq. (5).

The mean shortest age of the records for a given t is, evidently, $\langle l_{\min} \rangle = \int_0^\infty G(u,t)\,du$, — where $G(u,t) = \mathrm{Prob}[l_{\min} > u]$ is the cumulative distribution of the short-

est age. The Laplace transform of $\langle l_{\min}\rangle$ is given by

$$\int_0^\infty \langle l_{\min}\rangle\, e^{-st}\, dt = \int_0^\infty du\, \frac{\int_u^\infty q(l)\, e^{-sl}\, dl}{1-\int_u^\infty f(l)\, e^{-sl}\, dl}, \quad (13)$$

where the integrand — that expresses $\int_0^\infty e^{-st} G(u,t)\, dt$ — is obtained by taking a Laplace transform of Eq. (6) with respect to t, integrating over $\{l_i\}$ in (u,∞) and summing over M. The asymptotic behaviour of $\langle l_{\min}\rangle$ is extracted by analysing the leading singularity near $s=0$ in the above equation — that yields $\langle l_{\min}\rangle \sim \tau_0\big[\Gamma(1-\alpha/2)\big]^{-1}(t/\tau_0)^{1-\alpha/2}$ for large t.

Figure 3 compares the analytical asymptotic expressions of the averages mentioned above, with the respective values obtained from a numerical simulation of a CTRW with $\alpha = 2/3$.

In conclusion, we have shown that the statistics of records for a correlated time series generated by a CTRW display rather rich behaviour. We expect the results to be relevant to a wide class of problems. For example, CTRW has been recently used to model various time series that occurs in finance and economics [24], and hence our results would be useful in analysing those financial time series. For various contexts (e.g., in regard to the issues of "climate change with a warming trend"), it may be interesting to extend the study to non-symmetric $\phi(\xi)$ — using the generalised Sparre-Andersen theorem (see Ref. [29]). It is interesting to mention that for the particular case of the Cauchy distributed jumps with a constant drift μ — i.e., the jumps are drawn from a non-symmetric Cauchy PDF $\phi(\xi) = \pi^{-1}a/[a^2+(\xi-\mu)^2]$ — one obtains similar results, namely, $\alpha/2$ in the results of this paper is replaced by $\alpha[1/2+\pi^{-1}\tan^{-1}(\mu/a)]$ in that case[1]. Finally, it is useful to compare the statistics of the number of records of the CTRW with that of a time series of IID random entries drawn from $p(x)$ in continuous time with the waiting time between successive entries drawn from $\rho(\tau)$. In the latter case, it turns out that the distribution of the number of records $P(M,t)$ does not depend on $p(x)$ and for large t, it approaches a Gaussian around its mean $\langle M\rangle \sim \alpha \ln(t/\tau_0)$ with a variance $\langle M^2\rangle - \langle M\rangle^2 \sim \alpha \ln(t/\tau_0)$ — therefore, the qualitative features do not change drastically as one changes α. In contrast, for the CTRW we have found that the qualitative behaviour depends crucially on the parameter α.

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[1] Details will be published elsewhere.